Name:

Spring 2018 Math 245 Final Exam

Please read the following directions:
Please write legibly, with plenty of white space. Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. You may use a single page of notes, but no calculators or other aids. This exam will last 120 minutes; pace yourself accordingly. Please try to keep a quiet test environment for everyone. Good luck!

Problem	Min Score	Your Score	Max Score
1.	5		10
2.	5		10
3.	5		10
4.	5		10
5.	5		10
6.	5		10
7.	5		10
8.	5		10
9.	5		10
10.	5		10
11.	5		10
12.	5		10
13.	5		10
14.	5		10
15.	5		10
16.	5		10
17.	5		10
18.	5		10
19.	5		10
20.	5		10
Total:	100		200

REMINDER: Use complete sentences.

Problem 1. Carefully define the following terms:

a. floor

b. tautology

c. Trivial Proof theorem

d. modus ponens

Problem 2. Carefully define the following terms:

a. well-ordered

b. equicardinal

c. irreflexive

d. reflexive

Problem 3. Carefully define the following terms:

a. left-total

b. left-definite

c. surjection

d. identity function

Problem 4. Carefully define the following terms:

a. Modular Division theorem

b. Chinese Remainder theorem

c. Sperner's theorem

d. Order-Extension Principle

For problems 5 and 6, take ground set $S = \{3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ with divisibility relation $|$.

Problem 5. Draw the Hasse diagram for partial order $|$ on S.

Problem 6. Find, with justification, the height and width of the partial order $|$ on S.

Problem 7. Let $S = [0, 1]$, an interval in $\mathbb{R}$. Find a relation on S that is both a partial order and a function.

Problem 8. Let S be the set of all propositions (simple and compound). Consider the equivalence relation R on S defined as xRy if x is logically equivalent to y. Find three elements of $[p]_R$.

Problem 9. Prove the "Hypothetical Syllogism" theorem: Let p, q, r be propositions. Prove that $(p \to q), (q \to r) \vdash (p \to r)$.

Problem 10. Prove "Bernoulli's Inequality": Let $x \in \mathbb{R}$ with $x > -1$. Prove that $\forall n \in \mathbb{N}_0$, $(1 + x)^n \geq 1 + nx$.

Problem 11. Let a_n be a sequence. Prove that $a_n = \Theta(a_n)$.

Problem 12. Let R be a transitive relation on S. Prove that $R \circ R$ is transitive.

Problem 13. Find all integers $x \in [0, 18)$ satisfying the modular equation $15x \equiv 6 \pmod{18}$.

Problem 14. Define relation R on $\mathbb{Z}$ via $R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : 4|(a + b)\}$. Prove or disprove that R is an equivalence relation.

For problems 15 and 16, consider relation R on $\mathbb{N}$ via $R = \{(a, b) \in \mathbb{N} \times \mathbb{N} : b^2 \leq a < (b+1)^2\}$.

Problem 15. Prove that R is right-definite.
Note: do not assume that R is a function, and do not use problem 16.

Problem 16. Assume that R is a function. Prove or disprove that R is a bijection.

Problem 17. Let f, g be functions on $\mathbb{R}$. Suppose that $f \circ g \circ f = id_{\mathbb{R}}$. Prove that f is a bijection.

Problem 18. Let R be a partial order on S, and let $T \subseteq S$. Prove that a least element in T is unique.

Problem 19. Let S be a set, with two equivalence relations R, R'. Prove that $R \cap R'$ is an equivalence relation on S.

Problem 20. Define $\mathbb{R}^+ = \{x \in \mathbb{R} : x > 0\}$. Prove the following:
$\forall \varepsilon \in \mathbb{R}^+ \ \exists \delta \in \mathbb{R}^+ \ \forall x \in \mathbb{R}, \ |x - 3| < \delta \rightarrow |x^2 - 9| < \varepsilon$.